Reproductivity means how much the generation is reproducing itself. New period measures of fertility considering the effect of sex and mortality.

- **GRR (Gross Reproduction Rate)** sums up the age-specific fertility rates for only females' births (similar to TFR, but the numerator is limited for females' births; it's approximately same as TFR multiplied by the proportion of females at birth).
- **NRR (Net Reproduction Rate; denoted by R_0, pronounced as [ɑr nɔːt])** is a GRR adjusted for mortality: by multiplying each female ASFR by the probability of survival from birth to that age before summing up. This idea has been originally developed by Alfred Lotka and is applied to analyze infectious disease transmission as basic reproduction number.
  - The probability of survival from birth to age x is l[x] of lifetables.
  - 5Lx, the person-years survived from exact age x to x+5 (in other words, the stationary population for each age-class associated with the life table), will be (l[x]+l[x+5])/2*5.
  - Then female births to women in stationary population can be calculated by the product as (female ASFR)x(5Lx). Sum of these gives NRR.
- The interpretation of NRR is the average number of daughters a woman would have during her reproductive years assuming fertility and mortality schedules to be fixed.
- If NRR<1, the population in the synthetic cohort will decrease after considerable years.
- To be noted, the decrease does not necessarily happen in any actual cohort. Postwar Europe experienced the rapid rise in period fertility, when the NRR caused misleading. That's why TFR is more popularly used than NRR.
- Alternative calculation of NRR is, GRR multiplied by the probability of surviving to the mean age of the age-specific fertility distribution (l[x] at mbar). It's a good approximation. About the timing of fertility, the mean age of childbearing (μ) has become commonly used after mbar. μ is also the weighted mean of mid-points of the age groups, as same as mbar, where the weights are not females ASFR but the births to the stationary population. μ is generally slightly less than mbar, because μ considers the effect of mortality.
- Median ages of the age-specific fertility distribution (m) and the mean ages of mothers giving births (Mbar) are sometimes used. When considering stable population instead of stationary population, mean length of generation can be considered (see, Chapter 11).

```
# R code for text [http://minato.sip21c.org/demography-special/Chapter9.R]
T9.1 <- data.frame(AG = 2:9*5,
 Women = c(4533, 3777, 3101, 2636, 2161, 1793, 1484, 1222)*1000,
 TotalBirths = c(9066, 747846, 1045037, 819796, 566182, 353221, 140980, 17108),
 FemaleBirths = c(4410, 363738, 508286, 398733, 275380, 171800, 68570, 8321))
# TFR
print(TFR <- sum(T9.1$TotalBirths / T9.1$Women * 5))
# GRR
print(GRR <- sum(T9.1$FemaleBirths / T9.1$Women * 5))
# alternative (approximate.  In this case, very close
# because the sex ratio at birth are almost same for
# all age groups of mothers)
TFR * sum(T9.1$FemaleBirths) / sum(T9.1$TotalBirths)
# Confirm the sex ratios for each age group of mothers
# If sex ratio at birth is unknown, "males are 1.05 times
# of females" is usually acceptable (expressed by multiplying 100).
T9.1$FemaleBirths/T9.1$TotalBirths
(SexRatio <- sum(T9.1$TotalBirths-T9.1$FemaleBirths)/sum(T9.1$FemaleBirths)*100)
# NRR, which requires mortality data.
FemaleASFR <- T9.1$FemaleBirths / T9.1$Women
FemaleASFRx <- c(FemaleASFR[-1], 0) + c(FemaleASFR[1], rep(0, 7))
T9.2 <- data.frame(EAG = 3:10*5,
 FASFR = FemaleASFRx,
 lx = c(0.755, 0.735, 0.709, 0.683, 0.657, 0.628, 0.596, 0.558)
)
T9.2$L5x <- (T9.2$lx + c(T9.2$lx[-1], NA)) / 2 * 5
T9.2$FBSP <- T9.2$FASFR * T9.2$L5x
```

```
print(NRR <- sum(T9.2$FBSP, na.rm=TRUE)) # NRR
T9.2$MidpAG <- T9.2$EAG + 2.5
# mean age of age-specific fertility distribution
print(mbar <- sum(T9.2$FASFR * T9.2$MidpAG) / sum(T9.2$FASFR))
lmbar <- T9.2$lx[T9.2$EAG==25]+
  (T9.2$lx[T9.2$EAG==30]-T9.2$lx[T9.2$EAG==25])/5*
  (mbar-25) # linear interpolation between l25 and l30
GRR*lmbar # very similar to NRR
mu <- sum(T9.2$MidpAG*T9.2$FBSP, na.rm=TRUE)/sum(T9.2$FBSP, na.rm=TRUE)
print(mu)

# R code for exercise [http://minato.sip21c.org/demography-special/Chapter9E.R]
T9E.1 <- data.frame(x = 3:10*5,
 FP = c(234.0, 185.7, 112.5, 86.7, 107.0, 122.0, 112.6, NA)*1000,
 BirthBS = c(3986, 23798, 27433, 12065, 7642, 2771, 354, NA),
 lx = c(0.97518, 0.97258, 0.96916, 0.96524, 0.96006, 0.95209, 0.94091, 0.92435))
# Calculate the Gross and Net Reproduction Rates for Hong Kong, 1973.
# The sex ratio at birth is not given, so that 1.05 should be assumed.
# Before calculation, it can be expected that NRR and GRR are not so different.
# The reason is lx being close to 1 until age 50.
# And, the population structure is strange (30-34 is smaller than 35-39 and 40-44).
# GRR
T9E.1$FASFR <- T9E.1$BirthBS*(100/205)/T9E.1$FP
sum(T9E.1$FASFR*5, na.rm=TRUE)
# NRR
T9E.1$L5x <- (T9E.1$lx + c(T9E.1$lx[-1], NA)) / 2 * 5
T9E.1$FBSP <- T9E.1$FASFR * T9E.1$L5x
sum(T9E.1$FBSP, na.rm=TRUE)
print(T9E.1)
# According to the result, NRR is only slightly less than GRR, due to low
# mortality as expected.
# The Hong Kong women would produce about 1.5 daughters if the fertility and
# mortality schedules in 1973 are experienced forever.
```