Chandola T, Coleman DA, Hiorns RW (1999) Recent European fertility patterns: Fitting curves to 'distorted' distributions. *Population Studies*, **53:** 317-329.

Background

- To compare fertility, a lot of indices can be used: TFR, mean age of childbearing, distributional measures based on birth order or parity, and **detailed distribution of the age-specific fertility curves**, which can be obtained as model parameters. Recent trends in European fertility may converge into high mode, median and mean (see, Figure 1). However, there are considerable variation, in both of timing and volume (see, Figure 2). UK and Irish showed marked hump in early ferility. This study aims to model these 'distorted' patterns.
- Several models of age-specific fertility are known to give excellent fits to data from populations with natural fertility, with transitional fertility, and with controlled fertility: Pearson Type 1 (equivalent to B-distribution) and Type 3 (equivalent to Γ-distribution), polynomial functionss, Gompertz relational models (Brass, 1977), and Coale and Trussell (1974) model. Hadwiger distribution (Hadwiger, 1940) is a relatively less-known alternative. B-distribution and Γ-distribution cannot fit well especially for younger ages. Modified Γ-distribution (Hoem *et al.*, 1981) fitted well to Danish data, but the parameters have no demographic meaning, so that is not appropriate for comparison. Gompertz relational model does not fit well.

Coale and Trussell

$$F(x) = G(x)F_m(x), \quad F_m(x) = Mn(x)\exp(m\nu(x))$$

where $G(a)$ is ever-married proportion at age x, $F_m(x)$ is age-specific **marital** fertility rate for age x, $n(x)$ is age-specific natural fertility rate for age x, $\nu(x)$ is age-specific downward discrepancy from natural fertility for age x ($n(x)$ and $\nu(x)$ are given as schedules in Coale and Trussell's paper), M is overall scale factor and m is degree of control of fertility relative to the standard fertility schedule. It gives good fit to overall age-specific fertility rates and parameters have *a priori* demographic meanings, but it has originally treated age-specific **marital** fertility rate with intentional control and its computation is daunting.

Hadwiger

$$F(x) = \frac{ab}{c}\left(\frac{c}{x}\right)^{3/2}\exp\left\{-b^2\left(\frac{c}{x}+\frac{x}{c}-2\right)\right\}$$

where $F(x)$ is age-specific fertility rate for age x, a, b, c are parameters without any *a priori* demographic meaning (However, the authors suggested demographic meaning in this paper). Hoem *et al.* (1981) showed a good fit of Hadwiger function to the fertility curves of modern populations. Simple Hadwiger model fit well to the fertility curves of Austria, Denmark, France and Sweden in 1994 (Figure 3) but not to those of UK and Irish Republic in 1994 because of excesses of fertility at younger ages (Figure 4).

A mixture of Hadwiger

The possibility is that some populations including UK and Irish Republic are composed of two somewhat different populations. Let the mixture parameter m,

$$F(x) = m\frac{b_1}{c_1}\left(\frac{c_1}{x}\right)^{3/2}\exp\left\{-b_1^2\left(\frac{c_1}{x}+\frac{x}{c_1}-2\right)\right\} + (1-m)\frac{b_2}{c_2}\left(\frac{c_2}{x}\right)^{3/2}\exp\left\{-b_2^2\left(\frac{c_2}{x}+\frac{x}{c_2}-2\right)\right\}$$

The data were obtained from Eurostat (not downloadable nor free, but cf. http://europa.eu.int/comm/eurostat/). The mixture model fitted to the fertility curves of UK and Irish Republic in 1994 well (Figure 4). The mixture parameter m determines the relative size of the two component distributions and the mixture 1 and mixture 2 almost correnpond to non-marital and marital fertility curve, respectively (Figure 5). It suggests that the hump in UK and Irish Republic could be caused by a large proportion of teenage fertility, most of non-marital. Mixture 1 underestimates non-marital portion in later ages, which is absorbed by the relatively larger levels of mixture 2 than real marital fertility curves.

Since UK and Irish Republic have by no means the highest **levels of non-marital fertility (illegitimacy ratio)** in Western Europe, the relationship was tested for other countries (Figure 6), which was relatively linear. However, most of the other higher illegitimacy ratio populations obeyed the simple (homogeneous) Hadwiger model (no need for mixture), because there marital and non-marital fertility curves were relatively identical and there was no exceptionally high teenage fertility (Figure 7). In Australia, non-marital fertility is lower but not notably hererogeneous because timing was almost same as marital fertility.

Norwegian marital and non-marital fertility curves in 1994 were distinct both in timing and volume. Combination of these may generate the hump in younger age, which may fit with the Hadwiger mixture model. Figure 8 shows the emergence of 'distorted' fertility in UK (evident since 1980s).

Figure 9 shows the relationship between illegitimacy ratio and the mixture parameter m in UK and Irish Republic (linear for both countries, increasing over time). The dip in m in 1990s for UK suggests the relationship being not straitforward.

Other parameters of the simple Hadwiger model have demographic interpretations. Figure 10 shows the significant linear relationship between $a \times \frac{b}{c}$ and age-specific marital fertility rate (b affects the maximum age-specific fertility rate). Figure 11 shows the strong correaltions between a and TFR ($a = 0.56 \times \text{TFR}$) and between c and the mean age of motherhood.

Extending these interpretations, c_1 and c_2 in the Hadwiger mixture model may reflect mean ages of motherhood for non-marital and marital groups, respectively. In UK and Irish Republic, c_2 continously increased and c_1 decreased in 1990s (Figure 12). Mean age of non-marital motherhood in UK and Irish Republic has increased over this period, so that the Hadwiger mixture model does not exactly correspond to the mixture of marital and non-marital fertility curves.

Discussion

- The simple Hadwiger model successfully fits to fertility curves in some of modern European countries. Averaging these curves may give the standard European fertility curve. The Hadwiger mixture model fits well to the 'humped' fertility curves in some other European countries. These fits make it possible that is comparison of fertility and its components among European countries and over time.
- There was strong correspondence between a, b, c and TFR, maximum age-specific fertility rate, mean age of motherhood, respectively. It is new finding because Hoem *et al.* complained no demographic meaning of Hadwiger parameters. Moreover, it suggests that the Hadwiger models may be re-interpreted by more traditional demographic terms (not given in this paper).
- The Hadwiger mixture model may be related to marital and non-marital fertility. The mixture term m partly determines the relative size of two distributions, which related to illegitimacy ratio both cross-sectionally and longitudinally (This **cannot** be found by traditional measures!). c_2 may represent the mean age of marital motherhood, but considerable attention must be paid to interpret this, because c_1 does not correspond to the mean age of non-marital motherhood.
- Good fits of the simple Hadwiger model to some heterogenous fertility are attributable to the relatively small proportion of non-marital fertility as in Austria or to a similarity of timing and volume of marital and non-marital fertilities as in Denmark or Sweden. Societies with fitting the simple Hadwiger model to their fertility due to the latter reason may suffer from 'Second Demographic Transition' (indicating the disappearance of different reproductive behavior by marital status).

Further application of the Hadwiger function (by M. Nakazawa)

The method presented in this paper can easily be applicable to the fertility of postwar Japan. In Japan, non-marital fertility is virtually ignorable, so that I applied the simple Hadwiger model to the Japanese data during 1950 to 1995 compiled from the record of the vital statistics and the census report. Fitting were excellent for any year (RMSE≤ 0.012) and the parameter a was closely correlated with TFR ($R^2 = 0.999$) and c was closely correlated with the mean age of motherhood ($R^2 = 0.96$) also in Japan. Thus the method is useful to analyze the fertility decline in postwar Japan.

The trends of three parameters were well approximated by the cubic functions ($R^2 \geq 0.9$). Projected TFR in 2000 and 2005 were 0.63 and -0.34, respectively. Is it clearly wrong, probably due to a being less than zero in cubic trend approximation ($R^2 = 0.91$). Then, how is the real trend? If only the data during 1960-1995 were used for cubic trend approximation ($R^2 = 0.95$), a keeps positive after 2005, and projected TFRs are 1.37, 1.44 and 1.61 in 2000, 2005, and 2010, respectively. The selection of data points is more or less arbitrary and unreliable, but it may have some implication for fertility projection.