How to design sample size?

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■ Textbook in English

■ Textbook in Japanese
  □ 永田靖 (2003) サンプルサイズの決め方．朝倉書店
  □ 新谷歩 (2011) 今日から使える医療統計学講座【Lesson 3】サンプルサイズとパワー計算．週刊医学界新聞, 2937 号
  http://www.igaku-shoin.co.jp/paperDetail.do?id=PA02937_06
Due to the availability of patients within graduate school terms, only 10 patients were investigated to test the specific null-hypothesis X that factor Y is not related with factor Z, and the null-hypothesis could not be rejected at a significance level of 5%.

Based on the consideration that the result of "not significant" is also to be published to avoid publication bias, a manuscript entitled as "No relationship in the patients ..." was written and submitted to a journal.

The referee judged as "No significance may be caused by the less sample size." (In other words, the statistical power was not enough.) This is the fatal fault of the study design. Usually the manuscript is rejected.

The student asked to a statistician of helpful advices, but the statistician can do nothing to help the one.

It's a kind of common tragedy for many students.
What can the student do?

- The study framework was the hypothesis testing.
- Statistical power may increase with the sample size, so that it could be possible to determine the sample size to achieve enough statistical power before the study.
- After the study, the student can do nothing but writing excuse: e.g. due to less availability of the patients, short of funding, tradition in that specific study field. Sometimes it will pass the peer review because the result can contribute to future meta-analysis.
A study that is too small may be unethical, since it is not powerful enough to demonstrate a worthwhile correlation or difference.

Similarly, a study that is too large may also be unethical since one may be giving people a treatment that could already have been proven to be inferior.

Many journals now have checklists that include a question on whether the process of determining sample size is included in the method section (and to be reassured that it was carried out before the study and not in retrospect).

The statistical guidelines for the British Medical Journal in Altman et al. (2000) state that: `Authors should include information on … the number of subjects studied and why that number of subjects was used.'
A cynic once said that sample size calculations are a guess masquerading as mathematics. To perform such a calculation we often need information on factors such as the standard deviation of the outcome which may not be available. Moreover the calculations are quite sensitive to some of these assumptions.

Any study, whatever the size, contributes information, and therefore could be worthwhile and several small studies, pooled together in a meta-analysis are more generalisable than one big study.

Often, the size of studies is determined by practicalities, such as the number of available patients, resources, time and the level of finance available.

Studies, including clinical trials, often have several outcomes, such as benefit and adverse events, each of which will require a different sample size.
Not necessary cases

- Qualitative studies / Case report
- Small survey / pilot study
  - In descriptive study, usually previous information about the measures is unavailable, so that the sample size calculation is impossible.
  - Rules of thumb: at least 12 individuals in each group
    - List the main cross tabulations that will be needed to ensure that total numbers will give adequate numbers in the individual tables cells.
Two kinds of study

- Testing the null-hypothesis always requires the sample size calculation before the study (already explained).
- Exploring the hidden hypothesis or describing estimates with 95% confidence intervals may not always require the sample size calculation, but power analysis (to evaluate sampling adequacy) after the study is possible.

In the exploratory or descriptive studies

- Prevalence estimates from small samples will be imprecise and may be misleading. For example, when we wish to get the prevalence of a condition for which studies in other settings have reported a prevalence of 10%. A small sample of, say, 20 people, would be insufficient to produce a reliable estimate since only 2 would be expected to have the condition and ±1 would change the estimate by 5%.
- Sample size calculation determine the number of subjects needed to give a sufficiently narrow confidence intervals.
Examples in exploratory studies

- Values are obtained from previous studies in advance.
  - When we would like to **estimate a mean**, the following 3 values are needed.
    - The standard deviation (SD) of the measure being estimated
    - The desired width of the confidence interval (d)
    - The confidence level (usually 90, 95, or 99 %; 1-alpha)
  - Necessary number of samples (n) is obtained by:
    \[ n = \text{qnorm}(1-\text{alpha}/2)^2 \times 4 \times \text{SD}^2 / d^2 \]
  - (e.g.) Suppose we wish to estimate mean systolic blood pressure in a patient group with a 10mmHg-wide (or 5mmHg-wide) 95% confidence interval. Previous work suggested using a standard deviation of 11.4.
    \[ n = 1.96^2 \times 4 \times 11.4^2 / 10^2 = 19.97... = 20 \]
    \[ n = 1.96^2 \times 4 \times 11.4^2 / 5^2 = 79.88... = 80 \]
  - Doubling the precision needs quadrupling the sample size.

- Estimating proportions will be given in the next slide.
**Examples in exploratory studies (cntd.)**

- Required information from previous studies and study purpose to estimate proportion
  - Expected population proportion \( (p) \)
  - Desired width of confidence interval \( (d) \)
  - Confidence level \( (1-\alpha) \)

- Approximate equation to estimate the number of subjects needed
  \[
  n = \frac{\text{qnorm}(1-\alpha/2)^2 \times 4 \times p \times (1-p)}{d^2}
  \]

- (e.g.) Suppose we wish to estimate the prevalence of asthma in an adult population with the width of the 95% confidence interval 0.10, an accuracy of \( \pm 0.05 \). An estimate of the population prevalence of asthma is 10%.
  - \( p = 0.10, \ d = 0.10, \ \alpha = 0.05 \)
  - \( n = \text{qnorm}(0.975)^2 \times 4 \times 0.1 \times 0.9 / 0.1^2 = 1.96^2 \times 36 = 138 \)
Principles in hypothesis-testings

- What kind of information is needed?
  - Method of statistical test (including null-hypothesis)
  - Type I error (alpha error: probability to reject the true null-hypothesis, in other words, false positive)
  - Type II error (beta error: probability to fail to reject the false null-hypothesis / false negative) = 1 – statistical power
  - Expected values from previous studies
  - Minimum differences of clinical importance

- Equations are quite different by statistical tests (and by textbooks, because all of those are of approximation)
  - Compare means by t-test:
    \[ n = \frac{2(z_{\alpha} - z_{1-\beta})^2 \cdot SD^2 / d^2 + z_{\alpha}^2 / 4}{2} \]
  - Compare proportions by \( \chi^2 \) test:
    \[ n = \frac{(z_{\alpha/2} + z_{1-\beta})^2 \cdot \{p1(1-p1)+p2(1-p2)\}/(p1-p2)^2}{2} \]

- Usually special softwares (nQuery, PASS, PS) or general statistics softwares (SAS, SPSS, STATA, EZR, R, etc.) will be applied.
Suppose we wish to compare the mean increased degree of elbow flexion between stimulated and control patients.  
- 4 degree difference has clinical importance.
- Let alpha error 0.05 and statistical power 90%.
- SD of increase of elbow flexion is assumed as 5 degree.

Calculation by the previous equation:
- \[ n = \frac{2(z_\alpha - z_{1-\beta})^2 \cdot SD^2}{d^2} + \frac{z_\alpha^2}{4} \]
- \[ = 2*(-1.64-1.28)^2*5^2/4^2+(-1.64)^2/4 \]
- \[ = 27.3174 \approx 27 \]

Based on this, 26 treated patients and 25 control patients were measured. They showed increase in elbow flexion by 16±4.5 and 6.5±3.4, respectively. The mean difference was 9.5 (95%CI was 7.23 to 11.73), t-test resulted in \( t=8.43, df=49, p<0.001 \).

Typical description of this design and statistical results should be written as follows:
We designed the study to have 90% power to detect a 4-degree difference between the groups in the increased range of elbow flexion. Alpha was set at 0.05. Patients receiving electrical stimulation (n=26) increased their range of elbow flexion by a mean of 16 degrees with a standard deviation of 4.5, whereas patients in the control group (n=25) increased their range of flexion by a mean of only 6.5 degrees with a standard deviation of 3.4. This 9.5-degree difference between means was statistically significant (95%CI = 7.23 to 11.73 degrees; two-tailed Student's t test, t=8.43; df=49; p<0.001). (Lang and Secic, 2006, pp.47)

Here the expected standard deviation 5 nor applied equation is not clearly written (both seems implicit).
Usage of the PS

- PS: Power and Sample Size Calculator
- http://biostat.mc.vanderbilt.edu/twiki/bin/view/Main/PowerSampleSize
- Free Software
- Survival (logrank test), t-test, Regression1, Regression2, Dichotomous (chisq-test), Mantel-Haenszel are included.
- An example of description is given as text.
- Ratio of two groups can be specified as $m$. 

![Screenshot of Power and Sample Size Program](image)
Usage of EZR on Rcmdr

- EZR on Rcmdr is developed by Jichi Medical School
- Free software; latest version is 0.96 on 31 July 2011.
- Installation is very easy. Just click the downloaded executable installer. Messages are given in Japanese.
- Usage is also easy, but manual is not enough.
> power.t.test(delta=4, sd=5, sig.level=0.05, power=0.9)

Two-sample t test power calculation

    n = 33.82555
    delta = 4
    sd = 5
    sig.level = 0.05
    power = 0.9
    alternative = two.sided

NOTE: n is number in *each* group

The result is 34 (similar to PS)