Demography Special Lecture (10): Models of age structure 17 June 2021

The definition of "model"

(Oxford Advanced Learners' Dictionary)
SMALL COPY
1a copy of sth, usually smaller than the original object: a working model (= one in which the parts move) of a fire engine / a model aeroplane / The architect had produced a scale model of the proposed shopping complex.
DESIGN
2a particular design or type of product: The latest models will be on display at the motor show.
DESCRIPTION OF SYSTEM
3a simple description of a system, used for explaining how sth works or calculating what might happen, etc.: a mathematical model for determining the safe level of pesticides in food
EXAMPLE TO COPY
4 something such as a system that can be copied by other people: The nation's constitution provided a model that other countries followed.
5 (approving) a person or thing that is considered an excellent example of sth: It was a model of clarity. / a model student / a model farm (= one that has been specially designed to work well)—see also role model
FASHION
6a person whose job is to wear and show new styles of clothes and be photographed wearing them: a fashion model / a male model
FOR ARTIST
7a person who is employed to be painted, drawn, photographed, etc. by an artist or photographer

(Scientific meaning)
Some sort of **simplified representations of reality** where some aspects of the truth are omitted or assumed to be unchanging so that other aspects can be observed more clearly.=> classifiable to 2 groups (normative models and descriptive models)

Normative models have been used since long time ago, but descriptive models have become popular since 1950s. The oldest and most widely used normative models uses "synthetic cohort" like TFR, life table, NRR, stable population models.

Descriptive models are used for quite different purposes in demography. (1) Smoothing data (e.g., raw ASDRs may be irregular due to age-heaping or small sample sizes, so that smoothing can produce better estimates), (2) Fitting models to raw data for assessing data quality, (3) Imputing missing values or interpolating rough data (often used by historical demographers), (4) Forecasting (projecting) fertility, mortality, migrations in the future, (5) describing briefly the feature of a population (e.g., overall mortality levels can be described as life expectancy at birth using life table), (6) Evaluating the effect of changes in the determinants of mortality, fertility, and migration (e.g., Effect of rising ages at marriage or increasing contraceptive availability on fertility).

MODEL OF AGE STRUCTURE
Stable population (安定人口), Stationary population (静止人口), Intrinsic Rate of Natural Increase (内的自然増加率), Stable age distribution (安定人口分布) are the first target.

- Stable population theory, the findings by Euler (1760) and Lotka (1907): *If, in any population, mortality and fertility remain constant for a long peirod, and if there is no migration, then eventually a fixed age structure will develop and this "stable" age structure is completely independent from initial age structure*; where "stable" means the unchanging shape of an age distribution (total size of population is changeable).
- Special case of stable population, when growth rate is zero, is "stationary" population. The ${}_nL_x$ column in the life table is an example of stationary population.
- The size of the population consisting ${}_nL_x$ is T_0. ℓ_0 babies are born each year. Exactly same number dies because the size of population is unchangeable. CBR in the stationary population is "Intrinsic Birth Rate" (IBR): IBR = ℓ_0/T_0. It's exactly same as "Intrinsic Death Rate" (IDR).
- In non-stationary stable populations, IBR≠IDR. The difference (IBR−IDR) is "Intrinsic Rate of Natural Increase" (a.k.a. "True Rate of Natural Increase" or "Lotka's r"). Age structure is unchangeable and population growth rate is r, so that every age group must be growing at the same rate of r.
- NRR is average number of daughters a woman would have if she was to experience the given mortality and fertility rates: interpreted as a measure of the extent to which the population increases in a generation. **If the mean length of generation, *T*, is known, NRR = exp(*rT*)**. When we take logarithm of both side of the equation, ln(NRR) = rT, thus ***r*=ln(NRR)/*T***. In turn, if T is unknown and r and NRR are known, ***T*=ln(NRR)/*r***.

- There is no simple way to calculate *T*, but it can be approximated by the mean age of childbearing μ, so that $\boldsymbol{r}$**=ln(NRR)/**μ.
- *r* is quite different from CRNI (Crude Rate of Natural Increase = CBR−CDR).
- When *r* is positive, births in earlier years are fewer than now. The people aged 5 in 1974 is the survivors of the births of 1969.

<u>The process of calculation in Table 11.1</u>

ℓ_x for females and males, exact age *x*, number of years for each age group *n*, average age *y*, and M85+ (open-ended age groups' age-specific death rate) must be given as data.

1. Calculate the stationary population ${}_nL_x$ in normal way.
2. Calculate the intrinsic rate of natural increase, *r*.
3. Compute a column of the factors exp(-*ry*).
4. Multiply the stationary population by the factors to generate stable population.
5. The stable population is usually scaled to total population size as 10000.

<u>Doubling time</u>

When considering the rate of increase, it's an important question how long it would take for the population to be doubled, if the current increase rate were to continue.
If D is the doubling time and P is the population, then the population will increase exponentially to 2P by the formula, 2P = P*exp(rD). Thus, D = ln(2)/r.

* Please try Exercises 1-4.

See the R code given as **https://minato.sip21c.org/demography-special/Chap11.R**

(cf.) Intergenerational time intervals (Mean length of generation) in human population genetics
Trembly M, Vézina H (2000) New estimates of intergenerational time intervals for the calculation of age and origins of mutations. *Am. J. Hum. Genet.* 66: 651-658.

The time interval between two successive generations is not a fixed parameter. Among human populations, maternal age generally varies from 15 years to 50 years, whereas paternal age may reach higher limits. Because some age groups are more favorable to fertility, age difference between parents and their children is, in most cases, contained within a shorter interval. Though many studies were conducted, most authors use the mean value of 25 years, while others use the lower values of 20 years. Forster (1996) suggested a mother-daughter interval of 30 years in a sample of 17-19th century North German and Danish rural women. Cavalli-Sforza and Bodmer (1971) suggested a value of 29 years for female generation length.
The study given by Trembly and Vézina (2000) suggested mean value of 30 years is better estimate based on the data of BALSAC Population Register, a genealogical data of Quebec city. The calculation is the difference between marriage dates of parents and children, for mothers and fathers separately.

* See also, Chapter 9 "Stable and stationary models" of the book by Rowland DT (2003) Demographic methods and concepts. Oxford University Press, Oxford.

Table 9.1 Calculation of the intrinsic growth rate of a growing Western population, year 2000 (GRR=1.2, fe0=75)

Age group (A)	Midp-Age (B)	Female ASFRs per woman (C)	Probability of survival (D)	R0 working E (C x D)	R1 working F (B x E)
15-19	17.5	0.01326	0.97914	0.01298	0.22721
20-24	22.5	0.04324	0.97703	0.04225	0.95055
25-29	27.5	0.07812	0.97421	0.07611	2.09290
30-34	32.5	0.07113	0.97061	0.06904	2.24378
35-39	37.5	0.02906	0.96577	0.02807	1.05245
40-44	42.5	0.00506	0.95870	0.00485	0.20617
45-49	47.5	0.00013	0.94751	0.00012	0.00585
Total		0.24000		0.23341	6.77891
		GRR=1.2		R0=SUM(E)x5	R1=SUM(F)x5
				1.16707	33.89454
			ln R0	0.15450	
			R1/R0	29.04237	← mean length of generation
			r= (ln R0)/ (R1/R0-0.7 ln R0)	0.00534 -> 0.534% / yr	

Note: Equation for r is slightly different from the Newell's text (which doesn't have "-0.7 ln R0" in denominator).