Demography Special Lecture (11): Model life tables 24 June 2021 <minato-nakazawa@people.kobe-u.ac.jp>

Chapter 12 "Empirical model life tables", Chapter 13 "Relational model life tables", with parametric models

There are several approaches to model mortality schedules.

(1) Select the closest pattern to the actual mortality pattern among empirical patterns (Coale and Demeny, 1966, 1983 are famous) ==> Chapter 12
(2) Based on the linear relationships between logits of mortality of target and standard populations, setting 2 parameters to fit the target mortality pattern: Brass-model and Lee-Carter model ==> Chapter 13
(3) de Moivre, Gompertz, Gompertz-Makeham, Siler, Denny, Heligman-Pollard, ... many researchers suggested parameterized models to fit the target population's mortality pattern. ==> Not given in the text

Empirical models

Ledermann and Breas (1959) Most of the variation in mortality can be explained by: (1) The overall levels of mortality, (2) The ratio of child to adult mortality, (3) Old age mortality, (4) Infant mortality, (5) Sex differences. Amongst the overall level is most important.

- UN's (1955, 1956) collected 158 life tables for each sex => complex regression analysis => 24 model life tables for each sex (=> Table 12.1; https://www.un.org/en/development/desa/population/publications/pdf/manuals/projections/manual3/appendix.pdf).
- Coale & Demeny's (1966, 1983) Collected 326 male and 326 female real life tables, => 9 groups => 5 rejected due to inaccuracy or strongly affected by tuberculosis or small sample) => 4 families of life tables => calculate [nqx = a + be_{10}] for each age, sex, region separately, to adjust overall mortality model => 24 levels for each familes => 1983 revision raised upper limit of age from 80 to 100, using Gompertz model, and upper limit of females e0 changed from 77.5 to 80 as 25th level [North, South, East, West] (https://papp.iussp.org/sessions/papp103_s01/PAPP103_s01_070_010.html; https://doi.org/10.2307/3644567; https://www.un.org/development/desa/pd/data/model-life-tables; MORTPAK (free) https://www.un.org/en/development/desa/population/publications/mortality/mortpak.asp)
- UN's (1982) Based on 36 male and 36 female life tables collected from India, Iran, Kuwait, Israel, Tunisia, and developing countries in Central/Latin America, South-East Asia => 4 major patterns [Latin American, Chilean, South Asian, Far Eastern] and 5th [General] => 41 levels (e0 from 35 to 75 by 1 year) (https://www.un.org/en/development/desa/population/publications/mortality/model-life-tables.asp)
- UN's WPP (https://population.un.org/wpp/)
  - Basd on HMD (Human Mortality Database; https://www.mortality.org/), Lee-Carter model and Bayesian approach were applied. Now it's implemented in R packages (wpp2010, wpp2012, wpp2015, wpp2017, wpp2019, wppExplorer; https://bayespop.shinyapps.io/wpp2019explorer/).

Relational models

In R, using demography package is convenient.

- Brass relational two-parameter logit system's idea: Any population's life table (lx) can be linearly regressed [Y(x) = α+β Ys(x)] from Ys(x) = logit [0.5 log((1-ls(x))/ls(x))] of standard lx (=ls(x)) as Y(x). To fit this to actual lx (estimate α and β), l(2), l(3), l(5), l(45), l(50), l(55), l(60), l(65) were used. Taking averages of childhood points and adulthood points separately, and drawing the line through those 2 averages, then calculate 2 parameters.
- Zaba's (1979) 4 parameter model and Ewbank et al.'s (1983) 4 parameter model are improved (modified) version of Brass model.
- Lee-Carter (LC) model (1992) and its modified versions are *de facto* standard to forecast future life tables. Those can be considered as applied versions of relational models. Using **demography** package, **lca()** function gives LC model of mortality rates: Let m(x, t) the age (x) and time (t) specific mortality rate, **ln m(x,t) = α(x) + β(x)κ(t) + ε(x, t)**, where **Σβ(x)=1** and **Σκ(t)=0**.

Parameterized models

In R, **fmsb** package supports Gompertz-Makeham, Siler, Denny. **HPBayes** package supports Heligman-Pollard. There are many models. Denny's model has 3 parameters and fit well to any life tables's lx.

$$\ell(x) = \frac{1}{a\left(\frac{x}{105-x}\right)^3 + b\sqrt{\exp\left(\frac{x}{105-x}\right) - 1} + c\left\{1 - \exp\left(-2x\right)\right\}}$$

See the R code, will be given as **https://minato.sip21c.org/demography-special/Chap12.R** and **https://minato.sip21c.org/demography-special/Chap13.R**