

Chapter 12 "Empirical model life tables", Chapter 13 "Relational model life tables", with parametric models

There are several approaches to model mortality schedules.

- (1) Select the closest pattern to the actual mortality pattern among empirical patterns (Coale and Demeny, 1966, 1983 are famous) ==> Chapter 12
- (2) Based on the linear relationships between logits of mortality of target and standard populations, setting 2 parameters to fit the target mortality pattern: Brass-model and Lee-Carter model ==> Chapter 13
- (3) de Moivre, Gompertz, Gompertz-Makeham, Siler, Denny, Heligman-Pollard, ... many researchers suggested parameterized models to fit the target population's mortality pattern. ==> Not given in the text

### Empirical models

Ledermann and Breas (1959) Most of the variation in mortality can be explained by: (1) The overall levels of mortality, (2) The ratio of child to adult mortality, (3) Old age mortality, (4) Infant mortality, (5) Sex differences. Amongst the overall level is most important.

- UN's (1955, 1956) collected 158 life tables for each sex => complex regression analysis => 24 model life tables for each sex (=> Table 12.1; <https://www.un.org/en/development/desa/population/publications/pdf/manuals/projections/manual3/appendix.pdf>).
- Coale & Demeny's (1966, 1983) Collected 326 male and 326 female real life tables, => 9 groups => 5 rejected due to inaccuracy or strongly affected by tuberculosis or small sample) => 4 families of life tables => calculate  $[nqx = a + be_{10}]$  for each age, sex, region separately, to adjust overall mortality model => 24 levels for each families => 1983 revision raised upper limit of age from 80 to 100, using Gompertz model, and upper limit of females  $e_0$  changed from 77.5 to 80 as 25th level [North, South, East, West] ([https://papp.iussp.org/sessions/papp103\\_s01/PAPP103\\_s01\\_070\\_010.html](https://papp.iussp.org/sessions/papp103_s01/PAPP103_s01_070_010.html); <https://doi.org/10.2307/3644567>; <https://www.un.org/development/desa/pd/data/model-life-tables>; MORTPAK (free) <https://www.un.org/en/development/desa/population/publications/mortality/mortpak.asp>)
- UN's (1982) Based on 36 male and 36 female life tables collected from India, Iran, Kuwait, Israel, Tunisia, and developing countries in Central/Latin America, South-East Asia => 4 major patterns [Latin American, Chilean, South Asian, Far Eastern] and 5th [General] => 41 levels ( $e_0$  from 35 to 75 by 1 year) (<https://www.un.org/en/development/desa/population/publications/mortality/model-life-tables.asp>)
- UN's WPP (<https://population.un.org/wpp/>)
  - Based on HMD (Human Mortality Database; <https://www.mortality.org/>), Lee-Carter model and Bayesian approach were applied. Now it's implemented in R packages (wpp2010, wpp2012, wpp2015, wpp2017, wpp2019, wppExplorer; <https://bayespop.shinyapps.io/wpp2019explorer/>).

### Relational models

In R, using demography package is convenient.

- Brass relational two-parameter logit system's idea: Any population's life table ( $l_x$ ) can be linearly regressed [ $Y(x) = \alpha + \beta Y_s(x)$ ] from  $Y_s(x) = \text{logit} [0.5 \log((1-l_s(x))/l_s(x))]$  of standard  $l_x (=l_s(x))$  as  $Y(x)$ . To fit this to actual  $l_x$  (estimate  $\alpha$  and  $\beta$ ),  $l(2)$ ,  $l(3)$ ,  $l(5)$ ,  $l(45)$ ,  $l(50)$ ,  $l(55)$ ,  $l(60)$ ,  $l(65)$  were used. Taking averages of childhood points and adulthood points separately, and drawing the line through those 2 averages, then calculate 2 parameters.
- Zaba's (1979) 4 parameter model and Ewbank et al.'s (1983) 4 parameter model are improved (modified) version of Brass model.
- Lee-Carter (LC) model (1992) and its modified versions are *de facto* standard to forecast future life tables. Those can be considered as applied versions of relational models. Using **demography** package, **lca()** function gives LC model of mortality rates: Let  $m(x, t)$  the age ( $x$ ) and time ( $t$ ) specific mortality rate,  **$\ln m(x, t) = \alpha(x) + \beta(x)\kappa(t) + \varepsilon(x, t)$** , where  **$\Sigma\beta(x)=1$**  and  **$\Sigma\kappa(t)=0$** .

### Parameterized models

In R, **fmsb** package supports Gompertz-Makeham, Siler, Denny. **HPBayes** package supports Heligman-Pollard. There are many models. Denny's model has 3 parameters and fit well to any life tables's  $l_x$ .

$$l(x) = \frac{1}{a \left( \frac{x}{105-x} \right)^3 + b \sqrt{\exp \left( \frac{x}{105-x} \right) - 1} + c \{1 - \exp(-2x)\}}$$

See the R code, will be given as <https://minato.sip21c.org/demography-special/Chap12.R> and <https://minato.sip21c.org/demography-special/Chap13.R>