### **Cross tabulation**

- Comparison of proportions
  - Last week we learned the comparison of means or medians between the 2 quantitative variables
  - In the case of categorical variables, <u>comparison of proportions</u> can be applied (In the case of more than 2 groups, it's similarly applicable).
- Independence of 2 categorical variables
  - Making cross table, then test the null-hypothesis that the 2 variables are independent each other by <u>chi-square test</u> or <u>fisher's exact probability</u> <u>test</u>.
- The analyses above are essentially same.
- (cf.) "How large the effect of the difference of groups on disease occurrence is" can be measured by odds ratios, rate ratios, risk ratios, rate differences or risk differences (already learned)
- (cf.2) Association between 2 categorical variables can be assessed by <u>Pearson's contingency coefficients, phi coefficients, Cramer's V</u>, and <u>polychoric correlation coefficients</u>

### Comparison of 2 proportions

- If we compare the proportions of smoker between patients and controls, 40 among 100 and 20 among 100, respectively, type prop.test(c(40, 20), c(100, 100)) in the script window, and click [submit], then you can get the result, p-value=0.00337
- The way to conduct it in EZR menu, see below.



### Comparison of 2 proportions (cont'd)

- From the raw category data, for example, in the case of [survey] data of [MASS] package to compare the proportions of lefties between males and females, type prop.test(table(survey\$Sex, survey\$W.Hnd)) in the script window and click [submit].
- The way to conduct it in EZR menu, see below.

Statistical analysis Graphs and tables Tools Help	Original menu
Discrete variables	Frequency distributions
Continuous variables	Confidence interval for a proportion
Nonparametric tests	One sample proportion test
Survival analysis	Confidence interval for a difference between two proportions
Accuracy of diagnostic test	Confidence interval for a ratio of two proportions
Matched-pair analysis	Enter and analyze two-way table
Metaanalysis and metaregression •	Create two-way table and compare two proportions (Fisher's exact test)
Calculate sample size	Compare proportions of two paired samples (McNemar test)

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Sex=Male 10 108	

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### Chi-square test for independence

- In the chi-square test for independence,
  - First, make a cross table (contingency table) by the combination of 2 categorical variables: each number of cells can be denoted as n[i, j]. If each variable is binary, the table will be a 2x2 cross table.
  - Denote N1=n[1, 1]+n[2, 1], N2=n[1, 2]+n[2, 2], M1=n[1, 1] +n[1, 2], M2=n[2, 1]+n[2, 2], N=N1+N2=M1+M2
  - Calculate the expected number x[i, j] of each cell if row and column variables are independent.
    - x[1, 1]=N1\*M1/N, x[2, 1]=N1\*M2/N, x[1, 2]=N2\*M1/N, x[2, 2]=N2\*M2/N.
  - Calculate chi-squared as  $\chi^2 = \Sigma \{(n[i, j] x[i, j])^2/x[i, j]\}$
  - The value of  $\chi^2$  obeys chi-square distribution of d.f.=1.
- chisq.test(matrix(c(80, 20, 55, 45), 2, 2)) gives the result for the test for independence between smoking and lung cancer (pp.91-92).
- The way in EZR is similar to the comparison of proportions.

# Fisher's exact probability test for independence

- Chi-square test uses approximation (the combination can take only integer but the chisquare distribution is continuous) and thus is not appropriate for a small-sized sample.
- Fisher invented the direct calculation method to calculate the sum of the probabilities for the given combination and less plausible combinations under the condition of independence, where all marginal numbers are fixed, then the probability obeys hypergeometric distribution.
- This method for the same case is done by fisher.test(matrix(c(80, 20, 55, 45), 2, 2))
- The way in EZR is similar to the comparison of proportions, but check the [Fisher's exact test] of the dialogue.

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# Comparison of proportions among 3 or more groups

- Comparison of proportions among 3 groups
  - Schistosomiasis positive results of Kato-Katz fecal test were obtained from 60, 30, and 8 in 100 fishermen, 80 farmers, and 30 office workers, respectively, then prop.test(c(60, 30, 8), c(100, 80, 30))
    - For graph of positive proportions, try barplot(c(60, 30, 8)/c(100, 80, 30))
    - For pairwise comparisons, try pairwise.prop.test(c(60, 30, 8), c(100, 80, 30), p.adjust.method="fdr")
  - If we assume the tendency in the risk of subpopulations as 4:2:1, Cochran-Armitage test is applicable prop.trend.test(c(60, 30, 8), c(100, 80, 30), c(4, 2, 1))
- If the number of groups is more than 3, the way is similar.
- In EZR, directly entering the data of positive and negative numbers instead of positive and total, simple chi-square test is possible.

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# Association statistics (1): Pearson's contingency coefficients, and so on

- The extent of association between 2 categorical variables with 2 categories can be measured by the following indicators.
- Let's consider the relationship between smoking habits and lung cancer as already mentioned: The numbers of past smokers were 80 and 55 in Lung cancer patients and healthy controls, respectively.
  - Odds Ratio (rate and risk are unavailable in case-control study) is (80/20)/(55/45). It can be obtained by either of fisher.test(matrix(c(80, 20, 55, 45), 2, 2)) library(fmsb); oddsratio(80, 20, 55, 45)
  - The results are slightly different because fisher.test() uses maximum likelihood estimation, but both significant.
  - Other association statistics can be obtained by assocstats() function of vcd package, such as library(vcd); assocstats(matrix(c(80, 20, 55, 45), 2, 2))
  - Pearson's contingent coefficients and Cramer's V are applicable for the variables with 3 or more levels <u>without order</u>.

# Association statistics (2): Polychoric correlation coefficients

- When you like to test the association between non-binary categorical variables <u>with order</u>, polychoric correlation coefficient is better than rank correlation or contingency coefficients.
- Please consider the association of smoking habits and exercise in the [survey] data.
  - First, reorder factor levels of [Smoke] and [Exer] as [Smoke2] and [Exer2].
  - library(polycor) polychor(survey\$Smoke2, survey\$Exer2, std.err=TRUE)
  - It's different from polychor(survey\$Smoke, survey\$Exer, std.err=TRUE)



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# Repeated or Inter-rater agreement of categorical variables (Chap.13)

- When ordered or categorical variables were measured repeatedly or evaluated by multiple raters (observers), the result can be summarized as two-dimensional cross tabulation.
- However, common statistical testing for two-dimensional cross table like chi-square test or fisher's exact test is completely inadequate, because repeated or inter-rater measurements are clearly not independent.
- We have to test (1) the agreement significantly exceeds the expected one by chance, or (2) the agreement significantly worse than the expected one by chance.
  - (1) can be done by Kappa-statistics
  - (2) can be done by McNemar's test

### Kappa-statistics and McNemar's test

- Kappa statistics
  - Please assume the clinical test repeated 2 times, summarized as 2 by 2 cross table.
  - The agreement probability Po is (a+d)/(a+b+c+d).
  - If the agreement of the 2 test is perfect, b=c=0 (Po=1).
    When the tests completely disagree, a=d=0 (Po=0).
  - If the agreement is completely by chance, expected agreement probability Pe is {(a+c)(a+d)/(a+b+c+d)+(b+d) (c+d)/(a+b+c+d)}
  - Kappa statistics can be defined as (Po-Pe)/(1-Pe)
  - library(fmsb)
    Kappa.test(matrix(c(12, 2, 4, 10), 2, 2))
  - In EZR, [Statistical analysis]>[Accuracy of diagnostic test]>[Kappa statistics for agreement of two tests]
- McNemar's test
  - Evaluate the significant change of binary variable (pos/neg) between before/after intervention
  - The result is still 2 by 2 cross table.
  - X<sup>2</sup><sub>0</sub>= (b-c)<sup>2</sup>/(b+c), obeys chi-sq dist with d.f.=1
  - mcnemar.test(matrix(c(a, c, b, d), 2, 2))
  - By EZR, from raw data, see right.
  - Extended version is Bhapker's test (It's available as bhapker() in irr package).
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Test	Retest		
	Positive	Negative	
Positive	a (=12)	b (=4)	
Negative	c (=2)	d (=10)	

> .Table
Test1 (+) 12 4 Test1 (-) 2 10
> res <- NULL
> res <- epi.kappa(.Table, conf.
> colnames(res\$kappa) <- gettext
> res[1] \$kappa
est lower upper 1 0.5714286 0.2674605 0.8753967

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:	Metaanalysis and metaregression •	Create two-way table and compare two proportions (Fisher's exact test)
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	pos=17)	Compare proportions of more than two paired samples (Cochran Q test)